**Nuclear Decay and Medical Applications**

Now we’ll look at radioactive decay, and afterwards, medical applications and terminology.

Table

Description automatically generated

**Dynamics of Nuclear Decay**

When A gets too large, Z gets too large, and/or the numbers are unbalanced, then the nucleus is unstable and will decay over some lifetime. Example processes are:



Each unstable nucleus has a decay rate λ = 1/τ. This means that in any infinitesimal interval dt, the probability of decay is dt/τ. Might note that this makes decay a Poisson process. Suppose we have N such nuclei. Then a time dt later, we’d expect that Ndt/τ nuclei will have decayed. And so the number of nuclei will have changed by dN = -Ndt/τ. And so dN/dt = -N/τ = -Nλ. So our differential equation to solve is:



And so then solving for N(t), we get:



where N0 is the initial number of nuclei. Another way to write this formula is as follows. Let t1/2 be the half-life of the nuclei. This is the time it takes for half the nuclei to decay. In terms of this we may write:



We can write t1/2 in terms of λ by comparing the two formulas:



So,



Another important time-constant is the mean lifetime. Backing up, say a nucleus is present. What is the probability it won’t have decayed in the time span t? Well split the time span, t, into t/dt intervals dt. Then the probability it won’t decay within time span t is the probability it won’t decay in any of those intervals. And this is:



Now let’s take the dt → 0 limit. To facilitate this, we’ll take the ln of both sides,



So, upon exponentiation of both sides,



This makes some sense. Probability it won’t decay by t = 0 is 1, naturally, as time hasn’t started yet. But probability won’t decay by time t = ∞ is 0. Okay, well, the probability it will decay within interval (t, t+dt) is the probability it won’t have decayed up till time t, but then will decay in subsequent time interval dt. This is:



And since λ = 1/τ, we have:



We can now calculate the mean lifetime of a nucleus. So say it’s present at time t = 0. On average, how long will it last? This is:



So finally, we have:



The rate of decay, also called the activity, is designated R. It’s given by R = |dN/dt| = λN.



It’s measured in Becquerel (Bq) (one decay per second). Another measure is the Curie = 3.7×1010 Bq.

**Radioactive Dating**

*inorganic compounds* - ones not containing carbon

*minerals* - solid inorganic compounds, often contain metals

*crystal* - solid that grows in a polyhedral shape; the angles between the different faces is always the same for the same crystalline substance. The internal structure of a crystal is ordered.

So we can apply these concepts to work out the ages of rocks that crystalized at a common point in space and time. Consider an unstable parent nucleus P that decays into a daughter nucleus D1. Also consider a nucleus D2 which is an isotope of D1 (but not a decay product of P). Now let us imagine a molten soup containing some number of these nuclei NP, ND1, ND2. Since it is a fluid, we would imagine that the 3 elements would be distributed homogenously throughout the fluid. Then imagine that the fluid hardens into rock through some process, trapping the Np, ND1, ND2 in their homogenous distribution throughout the hardened fluid – now crystal. Now imagine that the solidified fluid breaks up into different pieces. These different pieces will all have the same ratio rD(0) = ND1(0)/ND2(0) owing to the initially homogeneous distribution. I would imagine that they would also have the same ratio rP(0) = NP(0)/ND2(0) initially. However, after time, since the rate of decay of the parent nuclei is proportional to the actual number of parent nuclei, each little crystal piece will come to have a different ratio rD(t) and rP(t) from the other crystal pieces. Let us try to develop a formula relating these two measurable quantities rD(t) and rP(t). We note that the number of parent nuclei + daughter nuclei in each crystal will remain constant for all time,



and that the number of daughter isotopes D2 in a given crystal will remain the same for all time as well.



Taking the ratio of these equations, we have:



Now solve this equation for Nd1(t)/Nd2(t).



Now recognize that Np(t) = Np(0)e-λt → Np(0) = Np(t)eλt. So we can say,



Furthermore, Nd2(0) = Nd2(t), so we can say,



Now we can combine like terms, to write:



Therefore we have



Suppose we take n crystals of common origin (and we implicitly assume that they therefore crystalized at the same time) and measure rDi(t0) and rPi(t0) for each one, where i is an index running from 1 to n, labeling each crystal. The daughter ratio rDi(t0), and parent ratio rPi(t0) of each crystal will be different due to the different *number* of parent isotopes originally solidified in the crystal. However for each crystal, rDi(t0) and rPi(t0) should be related by our equation. So if we plot rDi(t0) vs. rPi(t0), we should see a linear relationship with an intercept of rD(0), and slope eλt\_0 – 1. With the slope in hand, and knowledge of the decay constant λ, we can solve for the time of crystalization, t0.

**Example**

We would like to date a rock found at the bottom of the Grand Canyon. For these purposes, let’s note the following. 87Sr and 86Sr are stable isotopes of Strontium that are naturally found in the ratio 87Sr:86Sr = 0.70. This would be the ratio we presume to have been present in our rock at the top of its crystalization. Now we take a look at our rock and observe that its ratio is 87Sr:86Sr = 0.92. This is accounted for by the fact that our rock contains 87Rb, which beta decays into 87Sr with a half-life of about 48.8 billion years. In our rock, the ratio of 87Rb to 86Sr is about 2.6. What is the estimated age of our rock?

Well, we have:



**Example**

A geologist wishes to date some rock samples found in a dusty drawer in an old lab. The rock samples all had a common origin and contain Rb-87, Sr-87, and Sr-86 in varying amounts. It is presumed that at the time of crystalization, these three elements were mixed uniformly throughout the rock samples. This would change after crystalization, as Rb-87 decays into Sr-87 (Sr-87 and Sr-86 are themselves stable, however). Using a mass spectrometer, the geologist takes the samples and measures the present ratios of Sr-87:Sr-86 and Rb-87:Sr-86 for each one. She then plots the Sr-87:Sr-86 ratio on the y-axis, the Rb-86:Sr-86 ratio the x-axis, and finds a slope of 0.045. Given that Rb-87 has a half-life of 48.8 billion years, what age does she estimate for these rocks?

Well, we have:



The slope is:



So we’d estimate the age as:



**Radiocarbon dating**

We can use radiocarbon dating to date organic life in the following way. So cosmic rays from space (atomic nuclei, often) interact with the 14N in our atmosphere producing the isotope 14C, which, along with the regular 12C in our atmosphere, oxidizes immediately to form CO2, and then mixes homogeneously with the CO2 made out of 12C. Of course the 14C eventually (β?) decays to 14N with a half life of about 5730 years, and the carbon dioxide molecule is destroyed or maybe changes into NO2. But it is replaced by the newly formed ones out of the cosmic rays. So there is an equilibrium between these two processes and there remains a steady approximately 1 14C carbon dioxide molecule per every trillion 12C carbon dioxide molecules.

Carbon dioxide and its isotope are absorbed by plants in the process of photosynthesis, and oxygen is released. The carbon remains in the plants. Animals eat the plants and then they acquire the isotope as well. Each animal acquires the isotope in different amounts, but not different ratios. When the animal or plant dies, it stops absorbing the carbon dioxide, and hence its isotope, at least from the air; how about anywhere else? Thus the 14C decays to 14N, and isn’t replaced. So by measuring, the mass of the sample, you can get an idea of the amount of carbon dioxide of both types there is. Then you can use a radiation counter to determine how much 14C carbon dioxide there is in the sample, since the rate of change is proportional to that amount. From that, the age of the sample can be predicted, as we can see below.

So let r be the ratio NC14/NC12 (i.e., number of 14C’s divided by number of 12C’s). We know what r(0) is, because that should be what it is presently, in the atmosphere, about 1/1012, according to what we said above.



Now in the sample, 14C will decay to 14N, but the number of 12C remains the same. So we have:



But of course the number of 14C will exponentially decay according to:



And we presume to know λ, through experimental tests of that decay process. Now then let’s solve for the ratio r(t) at some later time. We have:



I think the typical way to get r(t) = NC14(t)/NC12(t) is to take a piece of it, vaporize it, and run it through a mass spectrometer. Then we’ll get NC14(t), and NC12(t) separately, and from these, calculate the ratio r(t). Then once we have r(t), and r(0), it’s simply to solve for t.

A few caveats…Cosmic ray production isn’t exactly constant so the 14C density in the atmosphere varies somewhat, though slightly. Even so, the procedure can be calibrated by using known dates, for example as with the bristlecone pines to shore up any problems. Given the recent industrial revolution, this method probably won’t be applicable in years to come given the higher concentrations of 14C being released into the atmosphere.

**Example:**

The half-life of 14C is 5730 years. So suppose that the activity of 14C in the atmosphere is 0.255 Bq per gram of Carbon. And suppose that the activity of the specimen is 0.0149 Bq per gram. Then what is the age of the sample?

Well, the activity of the sample can be written as:



(NC14 is number of 14C atoms) So the ratio of the activities will give us the ratio of the number of NC14 atoms. This ratio is:



Therefore we have,



And these are related through time via:



Therefore,



So now we need λ. We can get this from the half-life.



And so the time is:



**Example**

A team of scientists discover a new planet and want to find the age of the planet. They find that 40K , which has a half-life of 1.251×109 y, and 39K , which is stable, are both present on the planet. Their abundances are n40 and n39 , respectively, with n40 / n39 = 0.33. Assume both isotopes were equal in abundance when the planet was formed. What is the age of the planet?



So,



Yep.

**Example**

To date igneous rocks, we can use Potassium-Argon dating. Potassium-40 has a half-life of about 1.25 billion years, and its two decay products are Argon-40 and Calcium-40. When Potassium-40 decays it has an 11% chance of decaying into Argon-40, and an 89% chance of decaying into Calcium-40. When a rock sample is analyzed in a mass spectrometer, it is found that the ratio of Argon-40 to Potassium-40 is 0.27. What is the estimated age of the rock?

We can say,



We also have:



Taking the ratio,



Equating this to our observed ratio, we can solve for t:



So there.

**Medical Physics**

So now we’ll look at a few considerations about the medical applications of nuclear physics.

**Radiation Doses (basically energy density)**

The absorbed dose of radiation is the amount of energy absorbed by the tissue per unit mass of the tissue – it rather like energy density.



Common units are: 1 Gray (Gy) = 1 J/kg, and 1 rad = 0.01Gy.

**Radiation harmfulness**

But the harmfulness of the radiation depends on the type of radiation as well as the dose. Typically more massive radiation is more harmful than less massive radiation. The relative harmfulness (i.e. the *relative biological effectiveness* (RBE)) of the radiation is given below. The table below illustrates typical RBE’s

|  |  |
| --- | --- |
| Radiation | RBE |
| X rays and γ rays | 1 |
| Electrons | 1 |
| Protons | 5 |
| Slow neutrons | 5-20 |
| α particles | 20 |
| Heavy ions | 20 |

**Equivalent Dose**

Multiplying the Dose and the RBE gives the dose *equivalent*.



The units of Dose Equivalent depend on what units you use for Dose. If you use Gy, then the units of Dose Equivalent are Sievert (Sv). If you measure Dose in rad, then the units of Dose Equivalent are Rem. On other words:

Sv = Gy × RBE

Rem = rad × RBE

**Example**

It is estimated that one typically receives about 3.6 mSv per year. Supposing that this is entirely from 20 keV X-rays, how many of these X-rays does one absorb in a year? Well supposing a typical mass of 60kg, we’d have,



and so,



then dividing by the energy per X-ray, we get,

